

An Ideal Channel of Long Distance Entanglement in Spin Systems

Xiao-Qiang Xi^{1, 2}, T. Zhang³, R. H. Yue^{3, 4}, X. C. Xie², W. M. Liu²

¹*Department of Applied Mathematics and Physics,*

Xi'an Institute of Posts and Telecommunications, Xi'an 710061, China

²*Beijing National Laboratory for Condensed Matter Physics,*

Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China

³*Institute of Modern Physics, Northwest University, Xi'an 710069, China and*

⁴*Department of Physics, Ningbo University, Ningbo 315211, China*

We propose a scheme for using spin chain to realize an ideal channel of long distance entanglement. The results show that there has different entanglement in different Hilbert subspace, the anisotropic parameter Δ will frustrate the entanglement and the magnetic field affect the entanglement through changing the ground state, the boundary entanglement C_{1N} has the simplest expression in the simplest subspace and it only depend on the first item of the ground state, that item can be increased when a local magnetic field is introduced. Our propose can be handled easily because it only needs a uniform XX open chain initialized in the simplest Hilbert subspace and a bulk magnetic field that absent for the boundary qubits.

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Introduction. —Quantum entanglement has played an important role in modern physics, for example, it is used to test some fundamental questions of the quantum mechanics [1] and to act a central role in quantum information processing, like teleportation [2], super-dense coding [3], quantum computational speed-ups [4, 5], quantum cryptographic protocols [6, 7], one-way quantum computation [8] and so on. It is also widely used in sensitive interferometric measurements [9, 10, 11, 12] and studying strongly correlated quantum systems [13]. Especially, the ground-state entanglement can be related to quantum phase transition [14], Mott insulator-superfluid transition and quantum magnet-paramagnet transition. All of the applications of entanglement are closely dependent on how to produce it. Now, there are many physical systems suggested to realize entanglement [15, 16, 17, 18, 19, 20].

Spin chain is a nature candidate for producing pairwise entanglement. It has been used to quantum information processing [21, 22, 23, 24, 25, 26]. For the pairwise entanglement, most of works focused on the entanglement between the nearest pair, which is basic and will help us to deeply understand the character of entanglement, and looked the ways to control and maximize the entanglement, the effectively controlled factors include temperature, interchange coupling [23, 27], magnetic field and system impurity [28, 29, 30]. The ideal entanglement can be realized for the nearest pair, while the nearest pairwise entanglement is not enough for the practical applications because its short distance. Generally speaking, in solid system the entanglement between a pair decreases quickly as the increase of distance, so the works about non-nearest pairwise entanglement is scarce comparing with that of nearest, every non-nearest ideal entanglement will take great contribution to application of entanglement. There exist entanglement be-

tween the next-nearest-neighbor qubits in the transverse Ising model, but the maximal value of entanglement is about 4.3×10^{-3} [14]. A scheme is proposed for using a five-qubit open chain with magnetic and system impurities to realize a boundary entanglement with maximal value of $\frac{1}{2}$ [31]. Another scheme is proposed for realizing an ideal boundary entanglement in four-qubit open chain with symmetry interaction [32]. Venuti *et al.* [33] propose a scheme with a “strong-weak-strong-weak...” nearest interaction (dimerization) and a uniform next nearest interaction to realize a long-distance entanglement in spin system and further propose it to qubit teleportation and transfer. The starting point of those works is analyzing the affect of the interaction to the entanglement, their different lies in their analytical method.

The aim of this paper is to study how the interaction affect the non-nearest pairwise entanglement, especially the boundary entanglement in the open chain. We will analyze the corresponding results then use them as guidance to construct a practicable ideal channel of long-distance entanglement. The Hamiltonian of Heisenberg XXZ open chain with impurity is

$$H = \sum_{i=1}^{N-1} J_i (\sigma_i^+ \sigma_{i+1}^- + \sigma_{i+1}^+ \sigma_i^-) + \frac{\Delta}{2} \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^N B_i \sigma_i^z. \quad (1)$$

where J_i is the interaction between the i -th and $(i+1)$ -th qubits, Δ is the anisotropic parameter, B_i is the magnetic field, $\sigma^\pm = \frac{1}{2}(\sigma^x \pm i\sigma^y)$, $\sigma^x, \sigma^y, \sigma^z$ are the Pauli matrices.

The analytical tool in this paper is the concurrence theory [34, 35], C_{ij} denotes the pairwise entanglement between the i -th and j -th qubits, which ranges from 0 to 1 is monotonous to the entanglement.

Three-qubit Heisenberg XXZ open Chain in the uniform Magnetic Field. —When the interaction J_i and the magnetic field B_i are uniform, the eigenvalues of the system are $E_{0,7} = \Delta \mp 3B$, $E_{1,4} = \pm B$, $E_{2,5} = -\frac{\Delta \pm}{2} \pm B$, $E_{3,6} = -\frac{\Delta -}{2} \pm B$, where $\Delta_\pm = \Delta \pm \sqrt{8J^2 + \Delta^2}$

and the corresponding eigenvectors are

$$\begin{aligned}
|\psi_0\rangle &= |000\rangle, |\psi_7\rangle = |111\rangle, \\
|\psi_1\rangle &= (-|001\rangle + |100\rangle)/\sqrt{2}, \\
|\psi_m\rangle &= c_{m1}(|001\rangle + |100\rangle) + c_{m2}|010\rangle \quad (m = 2, 3), \\
|\psi_4\rangle &= (-|011\rangle + |110\rangle)/\sqrt{2}, \\
|\psi_n\rangle &= c_{n1}(|011\rangle + |110\rangle) + c_{n2}|101\rangle \quad (n = 5, 6), \quad (2)
\end{aligned}$$

where $c_{21} = c_{51} = \frac{\Delta_-}{\sqrt{2\Delta_-^2 + 16J^2}}$, $c_{22} = c_{52} = \frac{4J}{\sqrt{2\Delta_-^2 + 16J^2}}$,
 $c_{22} = c_{52} = \frac{4J}{\sqrt{2\Delta_-^2 + 16J^2}}$ $c_{31} = c_{61} = \frac{\Delta_+}{\sqrt{2\Delta_+^2 + 16J^2}}$ and
 $c_{32} = c_{62} = \frac{4J}{\sqrt{2\Delta_+^2 + 16J^2}}$.

Using the concurrence theory, the entanglement between the boundary qubits C_{13} can be obtained. The expression of $C_{13}(B, T, J, \Delta)$ is tedious, the numerical results show that the anisotropic parameter will frustrate the entanglement, and there exist a critical magnetic field for C_{13} , this phenomena comes from the change of ground state as the magnetic field variety. When $\frac{3\Delta + \sqrt{8J^2 + \Delta^2}}{4} < B$, the ground state is $|\psi_0\rangle$, $C_{13} = 0$; when $0 < B < \frac{3\Delta + \sqrt{8J^2 + \Delta^2}}{4}$, the ground state is $|\psi_2\rangle$, $C_{13} = 2c_{21}^2 = \frac{(\Delta - \sqrt{8J^2 + \Delta^2})^2}{8J^2 + (\Delta - \sqrt{8J^2 + \Delta^2})^2}$. For more clearly, we plotted the phase diagram of the ground state and the concurrence $C_{13}(B)$ for some certain Δ :

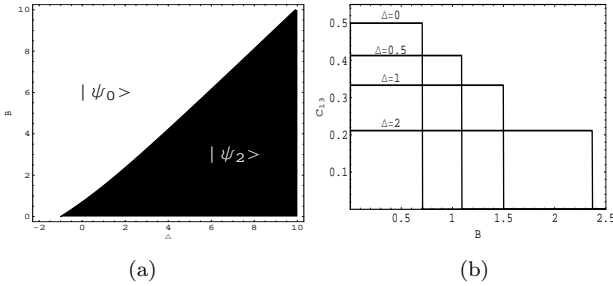


Figure 1. (a) The phase diagram of the ground state, the shading is $|\psi_2\rangle$ and the blank is $|\psi_0\rangle$; (b) the concurrence $C_{13}(B)$ for different Δ .

When the ground state is $|\psi_2\rangle$, C_{13} decreases as the increase of Δ , when $\Delta = 0$, $C_{13} = \frac{1}{2}$, when $\Delta = 1$, $C_{13} = \frac{1}{3}$, when $\Delta \gg J$, $C_{13} = 0$. i.e. the magnetic field can be used to switch “on” and “off” the entanglement. From (b) of Figure 1. we see that the interval of B (in which the entanglement exist) increases as the increase of Δ , that is to say, the large interval of entanglement existing is based on the decrease of entanglement. When $B = 0$, the ground states are $|\psi_2\rangle$ and $|\psi_5\rangle$ (duplicate degeneracy), $C_{13} = \max\{2c_{21}^2 - c_{22}^2, 0\} = 0$.

Four-qubit Heisenberg XXZ Open Chain in the uniform Magnetic Field. —As three qubit case, when J_i and B_i

are uniform the eigenvectors can be constructed as

$$\begin{aligned}
|\psi_0\rangle &= |0000\rangle, |\psi_{15}\rangle = |1111\rangle, \\
|\psi_m\rangle &= c_{m1}(|0001\rangle + e^{i\alpha_{m1}}|1000\rangle) \\
&\quad + c_{m2}(|0010\rangle + e^{i\alpha_{m2}}|0100\rangle) \quad (m = 1, 2, 3, 4), \\
|\psi_n\rangle &= c_{n1}(|1110\rangle + e^{i\alpha_{n1}}|0111\rangle) \\
&\quad + c_{n2}(|1011\rangle + e^{i\alpha_{n2}}|1000\rangle) \quad (n = 5, 6, 7, 8), \\
|\psi_k\rangle &= c_{k1}(|0011\rangle + e^{i\alpha_{k1}}|1100\rangle) \\
&\quad + c_{k2}(|0101\rangle + e^{i\alpha_{k2}}|1010\rangle) \\
&\quad + c_{k3}(|1001\rangle + e^{i\alpha_{k3}}|0110\rangle) \\
&\quad (k = 9, 10, 11, 12, 13, 14), \quad (3)
\end{aligned}$$

where the parameters c_{ij} , α_{ij} ($0, \pi$) are determined by $H|\psi\rangle = E|\psi\rangle$ and the normalization condition.

The results of four-qubit case is similar as that of three-qubit, the magnetic field change the entanglement through changing the ground state. The entanglement in the ground state is the maximal. We picked out the possible ground states and their eigenvalues: $|\psi_0\rangle$ ($E_0 = \frac{3\Delta}{2} - 4B$), $|\psi_1\rangle$ (the state in $|\psi_m\rangle$ with minimal eigenvalue $E_1 = -\frac{1}{2}(4B + J + \sqrt{5J^2 + 2J\Delta + \Delta^2})$), $|\psi_9\rangle$ (the state in $|\psi_k\rangle$ with minimal eigenvalue E_9 , which can be obtained for concrete Δ and J) and plotted the phase diagram of the ground state and the concurrence $C_{14}(B)$ for some certain Δ in Figure 2.

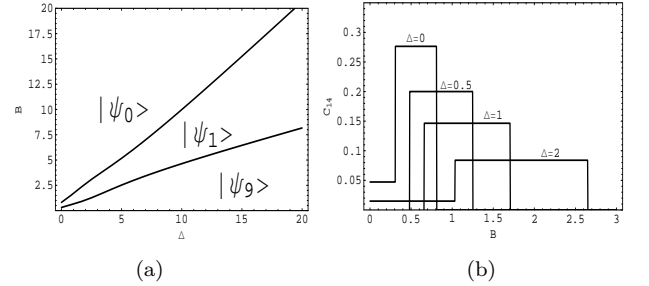


Figure 2. (a) The phase diagram of the ground state; (b) the concurrence $C_{14}(B)$ for different Δ .

The maximal value of C_{14} for different Δ and ground state are shown in Table 1.

Table 1. The entanglement C_{14} in the ground state, “GS(E)” instead of the ground state (eigenvalue).

| Δ | B | GS(E) | C_{14max} |
|----------|---|------------------------------|-------------|
| 0 | $0 \leq B < \frac{\sqrt{5}-1}{4}$ | $ \psi_9\rangle (-\sqrt{5})$ | 0.0472 |
| | $\frac{\sqrt{5}-1}{4} < B < \frac{\sqrt{5}+1}{4}$ | $ \psi_1\rangle$ | 0.2764 |
| | $\frac{\sqrt{5}+1}{4} < B$ | $ \psi_0\rangle$ | 0 |
| 0.5 | $0 \leq B < 0.48$ | $ \psi_9\rangle (-2.712)$ | 0 |
| | $0.48 < B < 1.25$ | $ \psi_1\rangle$ | 0.2 |
| | $1.25 < B$ | $ \psi_0\rangle$ | 0 |
| 1 | $0 \leq B < 0.66$ | $ \psi_9\rangle (-3.232)$ | 0 |
| | $0.66 < B < 1.70$ | $ \psi_1\rangle$ | 0.1464 |
| | $1.70 < B$ | $ \psi_0\rangle$ | 0 |
| 2 | $0 \leq B < 1.04$ | $ \psi_9\rangle (-4.372)$ | 0.0149 |
| | $1.04 < B < 2.65$ | $ \psi_1\rangle$ | 0.084 |
| | $2.65 < B$ | $ \psi_0\rangle$ | 0 |

When the ground state is $|\psi_1\rangle$, $C_{14} = 2c_{11}^2$ will decrease as the increase of Δ . When the ground state is $|\psi_9\rangle$, C_{14} is complex as the variety of Δ : $C_{14} = 0.0472$ if $\Delta = 0$; $C_{14} = 0$ if $0 < \Delta < 1$; C_{14} increases as the increase of Δ if $1 < \Delta < 7$, C_{14} decreases as the increase of Δ if $7 < \Delta$, the maximal value is about 0.063.

Heisenberg XX Open Chain with system impurity as an "ideal" entanglement channel. —From the conclusion of three- and four-qubit cases, we see that the boundary entanglement obtain its maximal value at $\Delta = 0$ then decrease as the increase of Δ . The uniform magnetic field can change the ground state and the corresponding entanglement. So in this section we no longer consider the anisotropy parameter Δ and the magnetic field B_i , XXZ model degenerate into XX model and we also suppose that all the degeneracy are eliminated.

For convenience, we use total spin to sign the ground state in different Hilbert subspace, if the ground state has k spin up and $N - k$ spin down, then total spin $S_T = \frac{N-k}{2} - \frac{k}{2} = \frac{N-2k}{2}$. For example, the total spin of $|\psi_0\rangle$, $|\psi_m\rangle$, $|\psi_k\rangle$, $|\psi_n\rangle$ and $|\psi_{15}\rangle$ in Eq. (3) is 2, 1, 0, -1 and -2 respectively. Because the symmetry of spin chain, we only consider the ground state with total spin $S_T = \frac{N}{2}, \frac{N}{2}-1, -\frac{N}{2}-2, \dots, \frac{N-2[\frac{N}{2}]}{2}$, the corresponding dimension are $C_N^N, C_N^{N-1}, C_N^{N-2} \dots C_N^{[\frac{N}{2}]}$ respectively. $S_T = \frac{N}{2}$ is a trivial subspace with dimension $C_N^N = 1$, in the other subspace, $S_T = \frac{N}{2} - 1$ with the minimal dimension $C_N^1 = N$ is the simplest, while $S_T = \frac{N-2[\frac{N}{2}]}{2}$ is the most complex. The entanglement of the system is bounded by the ground state entanglement of the subspace.

Without Δ and B , the only interaction left in Eq. (1) is the exchange hopping J_i , i.e. $H = \sum_{i=1}^{N-1} J_i(\sigma_i^+ \sigma_{i+1}^- + \sigma_{i+1}^+ \sigma_i^-)$. The case of $N=4$ is discussed thoroughly in Ref. [?]. Here we study it in the Hilbert subspace. Let $J_1 = J_3 = 1$, $J_2 = J$, for the ground state with $S_T = 1$ (i.e. $|\psi_m\rangle$ in Eq. (3), one spin up), $C_{14}(1) = 2c_{11}^2 = \frac{2(J-\sqrt{4+J^2})^2}{2(J-\sqrt{4+J^2})^2+2(2)^2} = \frac{J^2-J\sqrt{4+J^2}+2}{J^2-J\sqrt{4+J^2}+4}$, $C_{14}(1)_{max} = \frac{1}{2}$ when $J \rightarrow 0$; for the ground state with $S_T = 0$ (i.e. $|\psi_k\rangle$ in Eq. (3), two spin up), $C_{14}(0) = 2(|2c_{91}c_{92}| - c_{93}^2) = \frac{J\sqrt{J^2+4}-2}{J^2+4}$, $C_{14}(0)_{max} \rightarrow 1$ if J is large enough.

For $N = 5$, let $J_1 = J_4 = 1$, $J_2 = J_3 = J$. Calculations show that $C_{15}(\frac{3}{2}) = \frac{1}{2+4J^2}$ will get its maximal $\frac{1}{2}$ when $J \rightarrow 0$, the analytical result of $C_{15}(\frac{1}{2})$ can not be obtained, the numerical results show that $C_{15}(\frac{1}{2}) \rightarrow 1$ when $J \rightarrow \infty$.

When $N=6$, let $J_1 = J_5 = 1$, $J_2 = J_3 = J_4 = J$. The analytical result of C_{16} at the ground state with $S_T = 2, 1, 0$ can not be figured out. Numerical results show that $C_{16}(2)_{max} \rightarrow \frac{1}{2}$ when $J \rightarrow 0$; $C_{16}(1)_{max} \rightarrow 0.055$ when $J \approx 2$; $C_{16}(0)_{max} = 0.8$ when $J \rightarrow \infty$. If one let $J_1 = J_5 = 1$, $J_2 = J_4 = J$ and $J_3 = J^2$, $C_{16}(0)_{max} = 1$ when J is large.

Using this method, one can obtain the boundary en-

tanglement of N-qubit in the case of a matrix with order $\frac{(N-1)!}{[\frac{N}{2}]!(N-[\frac{N}{2}])!}$ can be manufactured. In fact only $N \leq 15$ (when $N = 15$, the matrix order is 429, great less than $2^{15} = 32768$) case can be calculated. Although the ideal entanglement spin channel with interaction $J_i = J_{N-i} = J^{i-1}$ can be realized, this propose has great disadvantage when N is large, because realizing a very strong interaction is very difficult. This channel is practicable only for a short distance entanglement.

A really ideal entanglement channel —A really ideal entanglement channel must be enough long and easy to manipulate. For the system with Eq. (1), the affect of Δ , J_i and the uniform magnetic field to the entanglement has been discussed in the above sections. They have no direct contribution to realize ideal entanglement channel. In the Hilbert subspace of N-qubit Heisenberg XX open, the simplest subspace is $S_T = \frac{N}{2} - 1$ except for the trivial cast $S_T = \frac{N}{2}$, such a subspace can be really manipulated when N is enough large, so we choose such a subspace as a candidate for studying the boundary entanglement ($C_{1N} = 2c_{m1}^2$), the most important thing is to distinguish the component $c_{m1}(|1\rangle + e^{i\alpha_{m1}}|N\rangle)$ ($m = 1, 2, \dots, N$) with the others, a good idea is to introduce the nonuniform magnetic field, in which the simplest case is to absent the magnetic field for the boundary qubits. So the Hamiltonian of the system can be written as

$$H = J \sum_{i=1}^{N-1} (\sigma_i^+ \sigma_{i+1}^- + \sigma_{i+1}^+ \sigma_i^-) + B \sum_{i=2}^{N-1} \sigma_i^z. \quad (4)$$

The initial state of the system is prepared as the first qubit to be spin up and the others spin down. Since the Hamiltonian commutes with the total spin component along the z direction, the relevant Hilbert subspace must be spanned by the states $|j\rangle = |0_1, 0_2, \dots, 0_{j-1}, 1_j, 0_{j+1}, \dots, 0_N\rangle$ with $j = 1, \dots, N$. So the eigenvectors of the system can be written as

$$|\psi_m\rangle = \sum_{j=1}^k c_{mj} (|j\rangle + e^{i\alpha_{mj}} |N+1-j\rangle), N = 2k \quad (5)$$

$$|\psi_m\rangle = \sum_{j=1}^k c_{mj} (|j\rangle + e^{i\alpha_{mj}} |N+1-j\rangle) + c_{m,k+1} |k+1\rangle, N = 2k+1 \quad (6)$$

where c_{mj} , α_{mj} (or π), $j = 1, 2, \dots, k$, $m = 1, 2, \dots, N$, are the parameters determined by $H|\psi\rangle = E|\psi\rangle$ and the normalization condition. Because $N = 2k$ is simpler than $N = 2k+1$, so we only consider even N case in the following discussion.

The maximal entanglement is determined by the ground state, suppose $|\psi_1\rangle$ is the ground state, then $C_{1N} = 2c_{11}^2$ will approximate to 1 if $\sum_{i=2}^k c_{1i}^2 \ll c_{11}^2$.

For $N = 2k$ case, one need to solve two $k \times k$ matrix,

they are

$$M_{\pm} = \begin{pmatrix} x_1 & J & 0 & \dots & 0 & 0 & 0 \\ J & x & J & \dots & 0 & 0 & 0 \\ 0 & J & x & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & x & J & 0 \\ 0 & 0 & 0 & \dots & J & x & J \\ 0 & 0 & 0 & \dots & 0 & J & x \pm J \end{pmatrix}_{k \times k} \quad (7)$$

with $x_1 = -(2k-2)B$, $x = -(2k-4)B$ and find its eigenvalues and eigenvectors. Only $k=2$ case can be calculated exactly, $C_{14} = \frac{(2B-J+\sqrt{4B^2-4BJ+5J^2})^2}{(2B-J+\sqrt{4B^2-4BJ+5J^2})^2+(2J)^2}$, which is great than 0.99 if $B/J > 5$, $C_{14} = 0.2764$ if $B = 0$. When $k \geq 3$, one can only obtain the numerical result of C_{1N} , fortunately, the numerical results show that the parameters in the ground state $|\psi_1\rangle$ satisfy $\frac{|c_{1i}|}{|c_{1,i+1}|} = \frac{2B}{J} = \beta$, $C_{1N} = 2c_{11}^2 = \frac{\beta^{2k}(\beta^2-1)}{\beta^2(\beta^{2k}-1)}$, which is great than 0.99 if $\beta > 10$. Details can be seen from Figure 3.

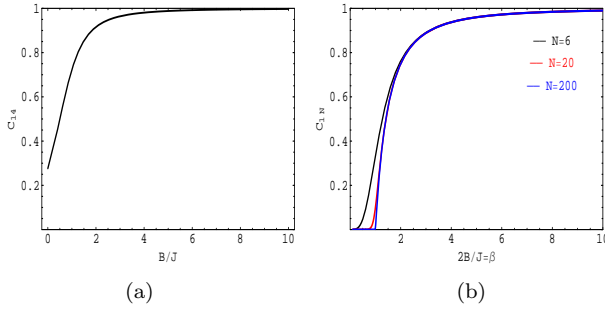


Figure 3. (a) The diagram of C_{14} with B/J ; (b) The diagram of C_{1N} with $2B/J = \beta$.

In theory, the length of ideal entanglement channel can be infinite when $2B/J$ is large enough. As the increase of N , the eigenvalue different between the ground state and excited states will be smaller and smaller.

Conclusions –The results for three- and four-qubit cases can tell us the affect of the anisotropic parameter Δ and B_i to the entanglement: Δ will frustrate the boundary entanglement; the affect of B_i to entanglement is to eliminate the degeneracy and change the ground state. In fact we can use same method to calculate $N(\leq 15)$ -qubit cases, the present results are still valid.

For the Heisenberg XX open chain with nonuniform symmetry interaction $J_i = J_{N-i} = J^{i-1}$ and J is large enough, the ideal entanglement can be realized in the most complex subspace, this conclusion has more theoretical meaning than its application's, while it is a good candidate if one needs a not too long distance entanglement.

For the Heisenberg XX open chain with uniform interaction J and a bulk magnetic field (the boundary qubits

are out of the magnetic field), the long distance ideal entanglement can be realized in the simplest Hilbert subspace. Our scheme needs two conditions, they are a uniform XX open with interaction J initialized in the simplest Hilbert subspace and a bulk magnetic field B absent for the boundary qubits. Under these conditions C_{1N} will be great than 0.99 if $B/J > 5$ for any even N . A uniform interaction chain is easier to realize than a chain with a “strong-weak-strong-weak-...” nearest interaction and a uniform next nearest interaction, in this aspect our scheme is simpler than that of Venuti *et al.*'s [33]. If our scheme can be realized in experiment, then teleportation in a solid system will become reality.

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- [1] J. S. Bell, *Speakable and Unsayable in quantum Mechanics: Collected Papers on Quantum Philosophy* (Cambridge 1987).
 - [2] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
 - [3] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
 - [4] P. W. Shor, SIAM Journal on Computing **26**, 1484 (1997).
 - [5] L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
 - [6] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
 - [7] D. Deutsch *et al.*, Phys. Rev. Lett. **77**, 2818 (1996).
 - [8] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
 - [9] A. N. Boto *et al.*, Phys. Rev. Lett. **85**, 2733 (2000).
 - [10] J. P. Dowling, Phys. Rev. A **57**, 4736 (1998).
 - [11] R. Jozsa *et al.*, Phys. Rev. Lett. **85**, 2010 (2000).
 - [12] J. J. Bollinger *et al.*, Phys. Rev. A **54**, R4649 (1996).
 - [13] J. Preskill, J. Mod. Opt. **47**, 127 (2000).
 - [14] T. J. Osborne and M. A. Nielsen, Phys. Rev. A **66**, 032110 (2002).
 - [15] D. Bouwmeester *et al.*, Nature **390**, 575 (1997).
 - [16] Q. A. Turchette *et al.*, Phys. Rev. Lett. **81**, 1525 (1998).
 - [17] A. Rauschenbeutel *et al.*, Science. **288**, 2024 (2000).
 - [18] F. Yamaguchi and Y. Yamamoto, Appl. Phys. A **68**, 1 (1999).
 - [19] Y. Makhlin, G. Schon and A. Shnirman, Nature **398**, 305 (1999).
 - [20] A. Sørensen *et al.*, Nature **409**, 63 (2001).
 - [21] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
 - [22] A. Imamoglu *et al.*, Phys. Rev. Lett. **83**, 4204 (1999).
 - [23] X. G. Wang, Phys. Rev. A **64**, 012313 (2001).
 - [24] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).
 - [25] V. Subrahmanyam, Phys. Rev. A **69**, 034304 (2004).
 - [26] T. J. Osborne and N. Linden, Phys. Rev. A **69**, 052315 (2004).

- (2004).
- [27] M. C. Arnesen *et al.*, quant-ph/0009060.
- [28] X. G. Wang, H. C. Fu and A. I. Solomon, J. Phys. A **34**, 11307 (2001).
- [29] X. Q. Xi *et al.*, Phys. Lett. A **297**, 291 (2002).
- [30] A. Abliz *et al.*, Phys. Rev. A **74**, 052105 (2006).
- [31] X. Q. Xi *et al.*, Acta. Phys. Sin. **55** 2036 (2006);
- [32] X. Q. Xi and W. M. Liu, Chin. Phys. **16**, 1858 (2007).
- [33] L. C. Venuti, C. D. E. Boschi and M. Roncaglia, Phys. Rev. Lett. **96**, 247206 (2006); *ibid* **99**, 060401 (2007).
- [34] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
- [35] S. Hill and W. K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997).